Modeling Spatial Means of Surfaces With Stratified Nonhomogeneity

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***Abstract*—In geosciences, one often needs to estimate the spatial means of surfaces representing physical attributes. Under certain conditions, this kind of estimation is often performed by a simple summation of a random sample or by some kind of a Kriging (spatial regression) technique. For example, the naive sample mean assumes that the sample is randomly distributed across space, which is a restrictive assumption with limited applicability in real-world situations (e.g., in the case of nonhomogeneous surfaces, the naive sample mean is a biased estimate of the actual surface mean). Kriging techniques can generate unbiased estimates for certain kinds of homogeneous surfaces but may be not appropriate in cases of stratified nonhomogeneity when the covariances exhibit considerable differences between different strata of the surface. In this paper, we extend the Kriging concept to study surfaces with stratified nonhomogeneity. The corresponding analytical formulas are derived, and empirical studies are performed that involve real-world and simulated data sets. Numerical comparative analysis showed that the proposed method performed well compared to other methods commonly used for the purpose of estimating surface means across space.**

***Index Terms*—Autocorrelation, Kriging, nonhomogeneity, sampling, spatial mean, spatial stratification, surface.**

I. Introduction

In GEOSCIENCES, a surface represents a spatially varying attribute (e.g., regional temperature, environmental pollution, cultivated land, landscapes, organic material in land, or population density), which means that the surfaces are interpreted according to the attributes being represented [1]–[4]. The areal average of a surface is defined as the integral of the attribute over the spatial region of interest and is usually estimated by designing a sampling survey or using an existing monitoring network [5]–[7]. The naive random sample mean (i.e., a simple averaged summation of the sample unit values) is often used to estimate the actual attribute average. This estimate would be unbiased if the sample is drawn randomly for both nonspatial data [8] and spatial data [9], [10]. However, the variance of the sample mean should be modified to account for spatial dependence (autocorrelation), since sampling techniques do not guarantee a minimum estimation variance [11], [12]. The unbiasedness and the minimum variance conditions of certain Kriging types considered in geographical sciences (e.g., Ordinary, Simple, and Block Krigings) [13] require that the surface is spatially homogeneous with a constant variance. Note that, in order to avoid confusion, the term "homogeneity" refers to the spatial features of an attribute, whereas the term "stationarity" is reserved to characterize the temporal variation of the attribute [14], [15].

As far as spatial data analysis is concerned, the modeling assumptions of sampling randomness and surface homogeneity are often not satisfied in practice [5], [7], [16]. This is due, among other things, to financial restrictions associated with spatial attribute sampling, the physical heterogeneity features of a surface, and the varying importance of different parts of the surface, e.g., there are usually more meteorological stations in densely populated areas than in less populated areas. The uneven spatial distribution of sampling and the spatial nonhomogeneity of the actual surface can generate a considerable bias between the naive sample mean and the population mean (because the available sample fails to match the structure of true surface), whereas the Kriging conditions (unbiasedness and minimum estimation error variance) are violated [13].

It is important to identify the type of nonhomogeneity characterizing the surface of interest, since some spatial estimation (mapping) methods are adequate for certain types of nonhomogeneity but not for others. There are different types and degrees of surface nonhomogeneity [17]–[19]. One type refers to an attribute with a spatially varying mean (nonhomogeneity in the mean), and another one refers to an attribute with a variance that changes from place to place (nonhomogeneity in the variance). Nonhomogeneity in the mean surfaces may be characterized by a global linear trend or by locally varying polynomial trends of higher degrees. Stratified nonhomogeneity characterizes a surface in which the covariances of different surface strata (subareas) exhibit considerable differences. Some nonhomogeneity features of a surface may be handled by the adaptive variogram method that localizes the associated parameters [20], whereas some others could be studied by adding a trend surface model in the Ordinary Kriging method [13]. However, certain nonhomogeneous surface features encountered in geographical sciences neither can be filtered out by a spatially continuous trend surface model nor can be modeled adequately by an adaptive variogram. Stratified nonhomogeneity is one of these features, which is why this paper proposes a method to study this kind of nonhomogeneous surfaces and generate adequate estimates of surface means across space by combining the Kriging concept with stratified statistics analysis.

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II. Surface Average or Means

A. Observed Surface Average

In geosciences, one often encounters a surface $y(s)$ that is a function of the spatial location (the s varies over an area $\Re$) under conditions of uncertainty. The surface $y(s)$ is characterized by stratified nonhomogeneity and is mathematically represented as a random field model with spatially varying mean and variance [21]–[23].

Assume that $\Re$ is decomposed into a set of spatially substrata $\{\Re_h, h = 1, \ldots, H\}$ within which the random field $y(s)$ is spatially homogeneous, i.e., [24]

$$E\left[y(s)|s \in \Re_h\right] = constant \tag{1}$$

where $E[\cdot]$ denotes the mean operator. In geographical sciences, the stratification may be achieved on the basis of expert knowledge about the particular kind of surface, familiarity with the underlying physical attributes, etc. [25]. The observed surface average over $\Re$ is defined as

$$\overline{Y}_\Re = \Re^{-1} \int\limits_{\Re} y(s) ds. \tag{2}$$

Clearly, one can also express the aforementioned average in terms of the substrata $\Re_h$ of $\Re$, viz.,

$$\overline{Y}_\Re = \Re^{-1} \sum_{h=1}^{H} \Re_h \overline{Y}_h \tag{3}$$

where the coefficient $\Re_h \Re^{-1} (h = 1, \ldots, H)$, defined by ξ_h, represents the areal percentage (or coverage ratio) of the hth stratum $\Re_h$, and

$$\overline{Y}_h = \Re_h^{-1} \int\limits_{\Re_h} y(s) ds \tag{4}$$

is the average surface within each individual stratum $\Re_h$.

B. Weighted Sample Mean

Assume that there are n_h sample points within each substratum $\Re_h$, so that the total number of sample points n over the study area $\Re$ is such that $\sum_{h=1}^{H} n_h = n$. The $\overline{Y}_h$ can be estimated by the weighted sample mean within a stratum $\Re_h$

$$\overline{y}_h = \sum_{i=1}^{n_h} w_{hi} y_{hi} \tag{5}$$

where y_{hi} denotes the sample value at site i within stratum $\Re_h$ and w_{hi} is the corresponding weight assigned to y_{hi}. In order for $\overline{y}_h$ to be an unbiased estimate of $\overline{Y}_h$, one must have

$$E[\overline{y}_h] = E[\overline{Y}_h] \tag{6}$$

or $\sum_{i=1}^{n_h} w_{hi} E[y_{hi}] = \Re_h^{-1} \int_{\Re_h} E[y(s)] ds$, which implies that

$$\sum_{i=1}^{n_h} w_{hi} = 1 \tag{7}$$

for all $h = 1, \ldots, H$, since $\int_{\Re_h} ds = \Re_h$, and $E[y_{hi}] = E[y(s)]$ due to the assumption of local homogeneity.

Let $\overline{Y}_\Re$ be estimated by a weighted strata mean

$$\overline{y}_\Re = \sum_{h=1}^{H} a_h \overline{y}_h. \tag{8}$$

For $\overline{y}_\Re$ to be an unbiased estimate of $\overline{Y}_\Re$, one needs that

$$E[\overline{y}_\Re] = E[\overline{Y}_\Re] \tag{9}$$

which implies that (Appendix A)

$$a_h = \Re_h \Re^{-1} = \xi_h. \tag{10}$$

Then, from (5), (8), and (10), one finds that the areal average of a surface is given by

$$\overline{y}_\Re = \Re^{-1} \sum_{h=1}^{H} \Re_h \sum_{i=1}^{n_h} w_{hi} y_{hi}. \tag{11}$$

Equation (11) is valid for a nonhomogeneous surface, in general. The mean surface average $\overline{y}_\Re$ of (11) is what one needs to calculate in many geographical science applications, and as such, it is the primary focus of this paper. Clearly, the determination of $\overline{y}_\Re$ requires the calculation of the coefficients w_{hi}'s ($h = 1, \ldots, H$ and $i = 1, \ldots, n_h$), which is the concern of the following section.

III. Spatial Estimation System for Stratified Nonhomogeneous Surfaces

In view of the aforementioned considerations, the spatial estimation problem is to find the weights w_{hi} ($h = 1, \ldots, H$ and $i = 1, \ldots, n_h$) in (11) that satisfy the unbiased condition (7) and minimize the mean squared estimation error

$$\sigma_\Re^2 = E[\overline{y}_\Re - \overline{Y}_\Re]^2. \tag{12}$$

As shown in Appendix B, the weights w_{hi} that satisfy the two conditions mentioned (unbiasedness and minimum mean squared estimation error) are given by the system of equations (case of stratified nonhomogeneity)

$$\left.\begin{array}{l} a_p \sum_{h=1}^{H} \sum_{i=1}^{n_h} a_h\, w_{hi} \operatorname{cov}(y_{hi}, y_{pj}) + \mu_p \\ \qquad = a_p \Re^{-1} \int\limits_{\Re} \operatorname{cov}\left(y_{pj}, y(s)\right) ds \\ p = 1, \ldots, H; j = 1, \ldots, n_p \end{array}\right\} \tag{13}$$

where μ_p's are the usual Lagrange multipliers of variational analysis [26], $\alpha_h = \Re_h \Re^{-1}$, and $\alpha_p = \Re_p \Re^{-1}$. $\operatorname{cov}(y_{hi}, y_{pj})$ denotes the covariance between the ith sample (s_i) within the hth stratum ($\Re_h$) and the jth sample (s_j) within the pth stratum ($\Re_p$). If $h = p$, $\operatorname{cov}(y_{hi}, y_{pj})$ expresses spatial dependence of surface values within the same stratum ($s_i, s_j \in \Re_h$), whereas, if $h \neq p$, $\operatorname{cov}(y_{hi}, y_{pj})$ expresses spatial dependence between different strata ($s_i \in \Re_h, s_j \in \Re_p$). In light of (13), the corresponding mean squared estimation error (12) can be expressed in terms of the coefficients w_{hi}'s and the multipliers μ_p's as follows:

$$\sigma^2_{\Re} = \Re^{-2} \int\limits_{\Re} \int\limits_{\Re} \mathrm{cov}\left(y(s), y(s')\right) ds ds' - \Re^{-1} \int\limits_{\Re} \sum_{h=1}^{H} \sum_{i=1}^{n_h} a_h w_{hi} \mathrm{cov}\left(y_{hi}, y(s)\right) ds - \sum_{h=1}^{H} \mu_h. \tag{14}$$

The covariance model $\mathrm{cov}(\cdot,\cdot)$ is determined by fitting an adequate theoretical model to the experimental covariance values calculated in terms of the attribute samples available. If other forms of physical knowledge are available, then it can also be used in the determination of the covariance model [27].

IV. Empirical Studies

The following empirical studies used three different data sets and eight theoretical covariance methods to calculate spatial surface means and to compare the performance of these methods. The performance was assessed by means of the corresponding surface mean squared estimation errors.

A. Data Sets

Two representative real-world data sets and one simulated data set were considered. Each data set was further stratified by expert knowledge [24], [25], as shown in Fig. 1. The two real-world data sets are stratified, respectively, according to the principle of maximizing the dispersion variance between strata and according to the principle of minimizing the dispersion variance within strata. The surfaces are characterized by stratified nonhomogeneity, in the sense that the covariances of different strata have considerable differences, and the differences were tested as being statistically significant.

Data Set No.1: Cultivated Land (Shandong Province, China) During the Year 2000—Fig. 1(a): The land cover data set is obtained from the Landsat TM/ETM remote sensing images during the year 2000. In order to reduce the cultivated land area into small grids, we converted the vector data into raster format with a grid size of 1 km × 1 km. The value of each raster grid is the cultivated land area in the grid's bound. The size of the data set is 655 × 439. The observed surface mean proportion of cultivated land was 659.419‰.

Data Set No. 2: Land Surface Temperature—Fig. 1(b): The data consist of the land surface temperature averaged during May 1–8, 2000. The data set was retrieved from MODerate-resolution Imaging Spectroradiometer (MODIS) images (MODIS/TERRA Land Surface Temperature/Emissivity 8-day L3 Global 1 km SIN GRID V004, http://elpdl03.cr.usgs.gov/pub/imswelcome/index.html). The spatial resolution was 927 m × 927 m, and the size of the data set was 706 × 706. The MODIS land surface temperature and emissivity (LST/E) products provided per-pixel temperature and emissivity values. Averaged temperatures are extracted in kelvins, with a day/night LST algorithm applied to a pair of MODIS daytime and nighttime observations. This method yields 1-K accuracy for materials with known emissivities. We converted Kelvin units to centigrade units. The observed surface mean value of land temperature is 29.015 °C.

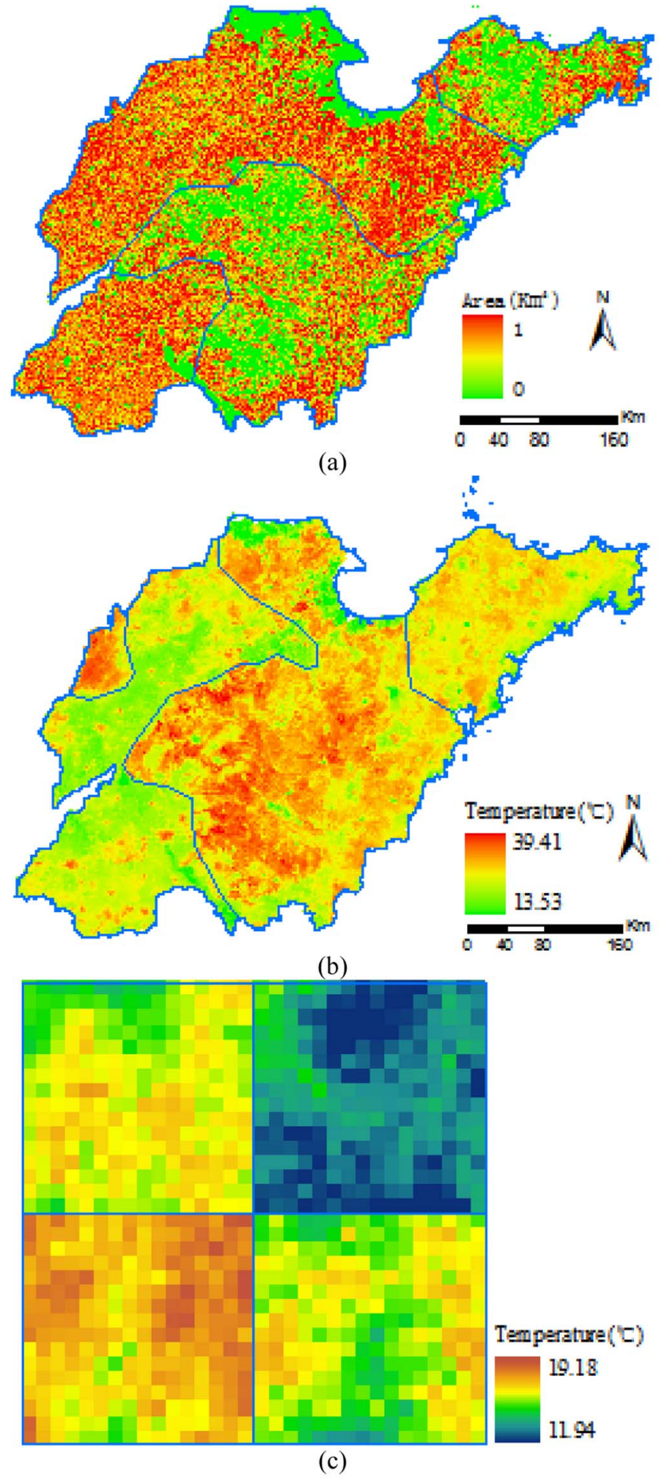

Fig. 1. (Blue lines) Three data sets and strata. (a) Cultivated land in Shandong during 2000. (b) Land temperature in Shandong during 2000. (c) Simulated temperature surface.

Data Set No.3: Simulated Temperature—Fig. 1(c): A Matlab toolbox (http://www.mathworks.de/matlabcentral/fileexchange/4849) was used to generate random numbers

TABLE I
MEAN AND DISPERSION VARIANCE OF EACH SURFACE STRATUM

Strata	Mean	Variance
First stratum	16.4791	0.58
Second stratum	13.8055	0.75
Third stratum	17.5453	0.71
Fourth stratum	16.2434	0.74
Average	16.0184	1.54

from a multivariate normal distribution and then rearrange them into an image. We have chosen a mean value to generate an image and calculate its covariance matrix between pixels. Four such 16×16 gridded images were generated, with mean and variance values given in Table I. Then, the four images were arranged to form a single image with size 32×32, with four strata. The observed population mean value of simulated temperature is 16.018 °C.

B. Methods for Estimating Spatial Means

Method 1: This is the proposed spatial mean estimation method, where the spatial dependence is expressed as follows: If $h = p$, each $\mathrm{cov}(y_{hi}, y_{pj})$ in (13) is determined by the theoretical model that is fitted to the samples of the specific stratum; if $h \neq p$, $\mathrm{cov}(y_{hi}, y_{pj})$ is determined by the global model fitted to the samples of all different strata. For example, the spherical covariance model fitted to the entire set of samples and the first stratum samples of cultivated land area in Shandong (China) are as follows, respectively:

$$c(h)=\begin{cases} 4018.7\left(1-\frac{3}{2\times 100\,796}r+\frac{1}{2\times 100\,796^3}r^3\right), & \text{for } r>0 \\ 16\,109.7, & \text{for } r=0 \end{cases} \quad (15)$$

$$c(h)=\begin{cases} 32\,854\left(1-\frac{3}{2\times 101\,060}r+\frac{1}{2\times 101\,060^3}r^3\right), & \text{for } r>0 \\ 96\,465, & \text{for } r=0 \end{cases} \quad (16)$$

where $r = |s - s'|$ is the distance between any pair of points s and s'.

Method 2: This is the proposed mean surface estimation method with zero between-strata dependence, where the spatial dependence is expressed as follows: If $h = p$, each $\mathrm{cov}(y_{hi}, y_{pj})$ in (13) is determined by the theoretical model that is fitted to the samples of the specific stratum; if $h \neq p$, $\mathrm{cov}(y_{hi}, y_{pj})$ is set equal to zero.

Method 3: This is the proposed mean surface estimation method in which the between-strata dependence is expressed by the pooled value minus two singles. In particular, if $h = p$, each $\mathrm{cov}(y_{hi}, y_{pj})$ is determined by the theoretical model that is fitted to the samples of the specific stratum; if $h \neq p$, $\mathrm{cov}(y_{hi}, y_{pj})$ is given by $[\mathrm{cov}(y_{hi}, y_{pj})_\Sigma - \mathrm{cov}(y_{hi}, y_{pj})_h - \mathrm{cov}(y_{hi}, y_{pj})_p]/2$, where $\mathrm{cov}(y_{hi}, y_{pj})_h$ and $\mathrm{cov}(y_{hi}, y_{pj})_p$ denote the covariances determined by the samples in the hth and the pth stratum, respectively, and $\mathrm{cov}(y_{hi}, y_{pj})_\Sigma$ denote the covariances determined by the samples in both the hth and the pth strata.

Method 4: Universal Kriging [13], [28]: This includes the unbiased estimation of unsampled surface values using the minimum mean squared estimation error criterion for trend surface analysis. Then, the samples and the estimates at the unsampled locations are added and averaged to obtain spatial means.

Method 5: Block Kriging [29], [30]: This is an unbiased estimate of the surface mean with minimum mean squared error estimation for the assumedly homogeneous surface.

Method 6: Ordinary Kriging [29], [31]: This includes the unbiased estimation of unsampled values with minimum mean squared estimation error for the homogeneous surface. Unlike the Universal Kriging method, the Ordinary Kriging assumes a homogeneous surface. As before, the samples and the estimates at the unsampled locations are added and averaged to obtain the spatial means.

Method 7: Spatial random sampling [9]: This takes spatial dependence (autocorrelation) into account and also calculates the variance of the corresponding spatial sample means.

Method 8: Simple random sampling [8]: The mean is estimated by simple summation of sample values. The variance of the sample mean is positively proportional to the dispersion variance of the samples and is negatively proportional to the number of sample units.

C. Comparative Performance of the Methods

For the estimation error variances of the spatial surface means estimated by the various methods discussed earlier and for the cultivated land, the land temperature and the simulated temperature data sets are shown in Figs. 2–4.

The numerical results show that the comparative performance of the proposed method was very good, i.e., it performed better or at least as good as the other methods. This is particularly valid in the case of the simulated temperature surface [Fig. 4(a) and (b)], where the spatial nonhomogeneity is well represented by a stratum [Table I and Fig. 1(c)]. The most significant differences between the methods happened when smaller numbers of samples were used. Clearly, the accuracy of all methods improved as the number of samples increased.

In the case of the two real-world data sets (cultivated land and land temperature), Universal Kriging seems to be the worst performer. This is probably due to the fact that the type of stratified nonhomogeneity characterizing these data sets is different from the usual nonhomogeneity assumed by the Universal Kriging method. Block Kriging is also a relatively poor performer in the case of the two real-world data sets, particularly when a smaller number of samples are considered. The simple sampling methods are the worst performers in the case of the simulated data set.

V. DISCUSSION AND CONCLUSION

A. Figures and Tables

In earth sciences, one often needs to estimate spatial surface means or averages [5], [9], [13], [31]–[34]. In this paper, a general method is considered for estimating the means of spatial surfaces characterized by stratified nonhomogeneity.

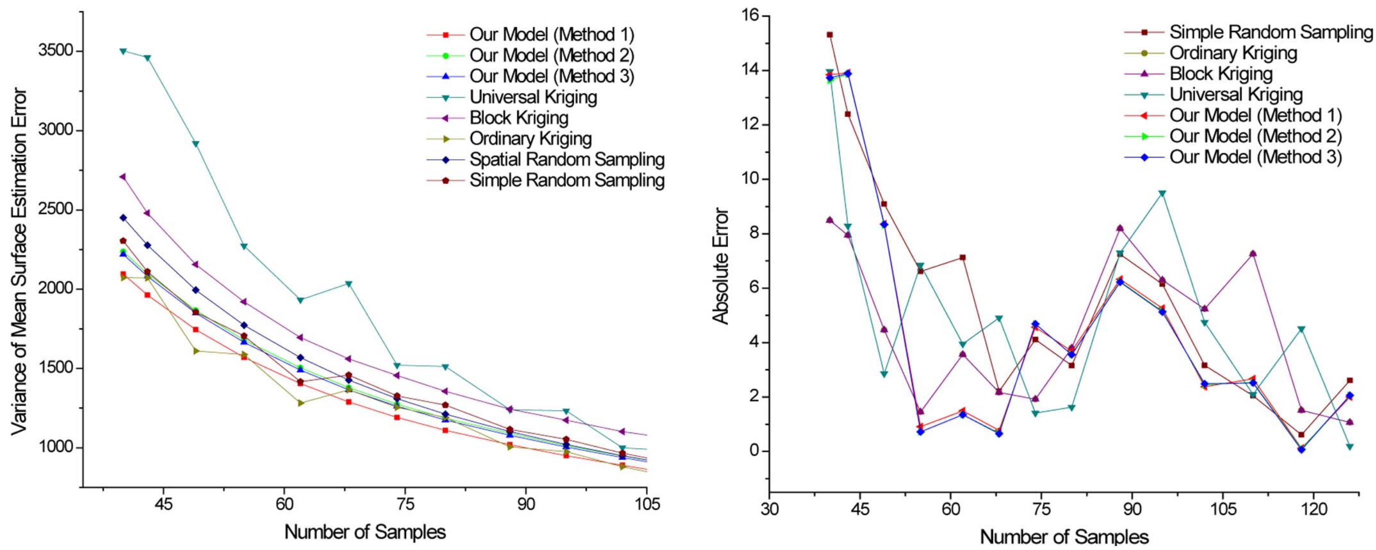

Fig. 2. (a) Comparison of theoretical estimation variance between the eight methods of spatial mean estimation, using cultivated land in the Shandong province (China) during 2000. (b) Comparison of the rooted square difference between the actual strata mean and the estimated one between the eight methods of spatial mean estimation, using cultivated land in the Shandong province (China) during 2000.

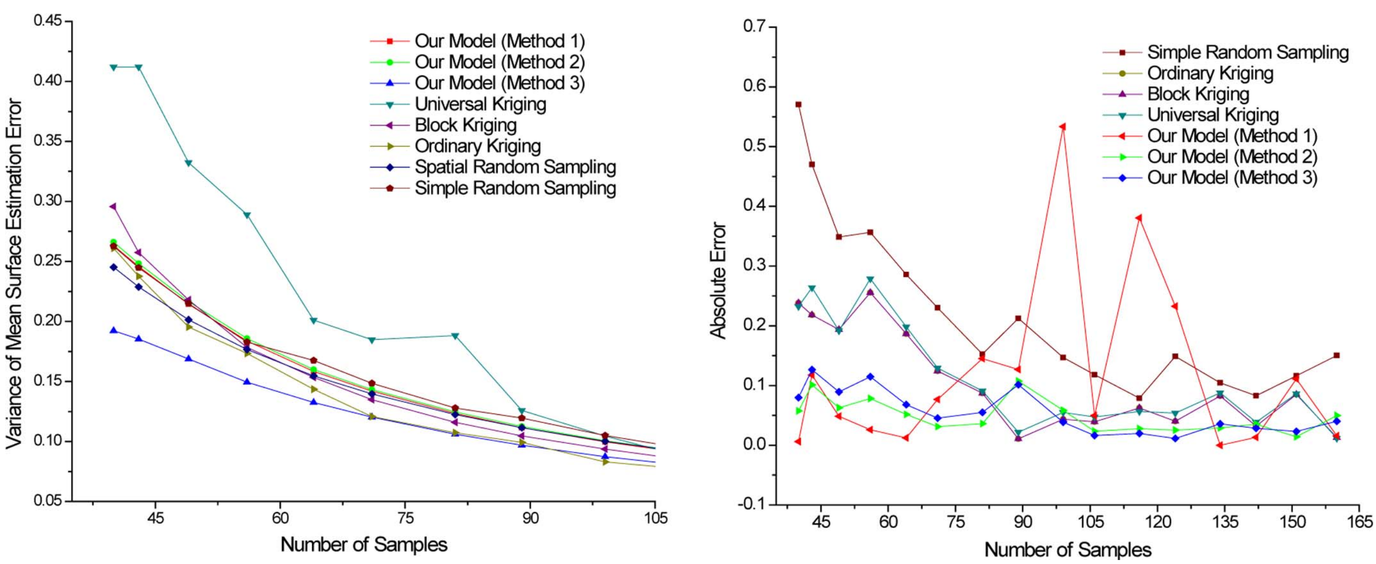

Fig. 3. (a) Comparison of theoretical estimation variance between the eight methods of spatial mean estimation, using MODIS temperature in the Shandong province (China) averaged during May 1–8, 2007. (b) Comparison of the rooted square difference between the actual strata mean and the estimated one between the eight methods of spatial mean estimation, using MODIS temperature in the Shandong province (China) averaged during May 1–8, 2007.

The method first decomposes a nonhomogeneous surface into smaller subareas (substrata) that are locally homogeneous in the mean. Then, the spatial mean of the surface and its variance are calculated by combining the techniques of Kriging and stratified sampling. The comparative performance of the proposed method involved three different data sets. Numerical analysis showed that the proposed method performed well compared to other methods commonly used for the purpose of estimating spatial surface means (including different kinds of Kriging and random sampling).

It should be noticed that the proposed model provides spatial mean estimates for nonhomogeneous surfaces, while Ordinary, Block, and CoKriging systems are relevant for homogenous surfaces. The Block Kriging (method 5) is not necessarily equivalent to the averaging of all the Kriging points within a block (method 6). Moreover, the objective of Block Kriging is to minimize the sample mean variance of the entire block σ^2_{BK}, which is not equivalent to a simple summation of the estimation variances at each site provided by Ordinary Kriging, $\Sigma\sigma^2_{\mathrm{OK}}$. This difference is also presented in the empirical example in Figs. 2–4.

In this paper, sample units across the nonhomogeneous population were properly weighted in order to minimize the global sample mean variance. Solving this optimization problem leads to (13), in which both the weighting and the pooled covariance are interwoven. The theoretical estimation variances are shown in Figs. 2–4, whereas Figs. 2–4 show the differences between the models' output and the actual strata mean. In practice, Figs. 2–4 are unavailable because the actual strata means are usually unavailable, which is why sampling is necessary.

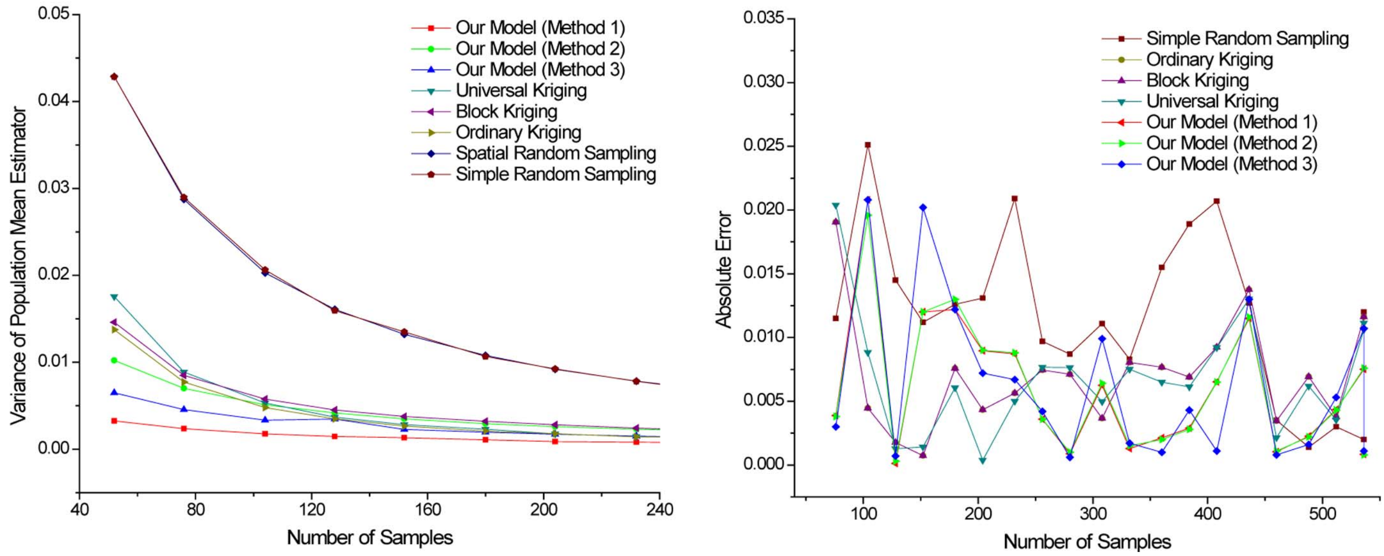

Fig. 4. (a) Comparison of theoretical estimation variance between the eight methods of spatial mean estimation, using the simulated temperature surface. (b) Comparison of the rooted square difference between the actual strata mean and the estimated one between the eight methods of spatial mean estimation, using the simulated temperature surface.

Theoretically, the spatial mean variance of this paper's models is minimized for nonhomogenous surfaces. To our knowledge, no other models aim at this objective, although there exist parallel efforts to handle the stratified surfaces, e.g., a possible solution to estimate the mean of a homogenous domain is to minimize the right-hand side of the (Ordinary or Universal) Kriging equations [35], [36]; these estimated means can then be weighted by their strata size [35]. That seems equivalent to the Method 2 in a case that the covariance between strata is set to be zero, i.e., the real surface is assumed perfectly stratified and its strata perfectly matched with the strata for sampling.

Empirically, the performance of the different models seems somewhat inconsistent across the different data sets, in the sense that the level of performance is not the same for each case. Aside from the randomness of the location of the sample sites, the different performances reflect the sensitivity of the different models to the surface features of spatial correlation and spatial heterogeneity. The spatial heterogeneity is well represented in Data set 3, and the three proposed methods perform well in this case. For nonhomogeneous surfaces, the proposed models are recommended. More specifically, Method 1 offers a universal setting for all types of surfaces. Method 2 is more appropriate when the real surface is well stratified and well matched by the sampling strata. Method 3 filters out the pure coregionalization variance, minus the variance of each of the strata from the pooled variance; it is stronger than well-stratified surfaces and less than continuous surfaces.

APPENDIX A

For $\overline{y}_{\Re}$ to be an unbiased estimate of $\overline{Y}_{\Re}$, one needs that (9) is satisfied. In view of (3) and (8), one finds that (9) can be written as

$$E\left[\sum_{h=1}^{H} a_h \overline{y}_h\right] = E\left[\Re^{-1}\sum_{h=1}^{H} \Re_h \overline{Y}_h\right] \tag{A1}$$

or

$$\sum_{h=1}^{H} a_h E[\overline{y}_h] = \sum_{h=1}^{H} \Re_h \Re^{-1} E[\overline{Y}_h] \tag{A2}$$

which implies that

$$a_h E[\overline{y}_h] = \Re_h \Re^{-1} E[\overline{Y}_h]. \tag{A3}$$

Due to homogeneity, $E[\overline{y}_h] = E[\overline{Y}_h]$, in which case (A3) yields (10).

APPENDIX B

In light of the unbiasedness condition (9) and assuming stratified nonhomogeneity, the mean squared estimation error (12) can be written as

$$\sigma^2_{\Re} = E\left[(\overline{y}_{\Re} - E[\overline{y}_{\Re}]) - \left(\overline{Y}_{\Re} - E[\overline{Y}_{\Re}]\right)\right]^2 \tag{B1}$$

or

$$\sigma^2_{\Re} = \mathrm{cov}(\overline{y}_{\Re}, \overline{y}_{\Re}) - 2\mathrm{cov}(\overline{y}_{\Re}, \overline{Y}_{\Re}) + \mathrm{cov}(\overline{Y}_{\Re}, \overline{Y}_{\Re}) \tag{B2}$$

where

$$\mathrm{cov}(\overline{y}_{\Re}, \overline{y}_{\Re}) = E\left[\overline{y}_{\Re} - E[\overline{y}_{\Re}]\right]^2 = \sum_{h=1}^{H}\sum_{i=1}^{n_h}\sum_{p=1}^{H}\sum_{j=1}^{n_p} a_h a_p w_{hi} w_{pj} \mathrm{cov}(y_{hi}, y_{pj}) \tag{B3}$$

$$2\mathrm{cov}(\overline{y}_{\Re}, \overline{Y}_{\Re}) = 2E\left[(\overline{y}_{\Re} - E[\overline{y}_{\Re}])\left(\overline{Y}_{\Re} - E[\overline{Y}_{\Re}]\right)\right] = 2\Re^{-1}\int_{\Re}\sum_{h=1}^{H}\sum_{i=1}^{n_h} a_h w_{hi} \mathrm{cov}\left(y_{hi}, y(s)\right) ds \tag{B4}$$

$$\mathrm{cov}(\overline{Y}_{\Re}, \overline{Y}_{\Re}) = E\left[\overline{Y}_{\Re} - E[\overline{Y}_{\Re}]\right]^2 = \Re^{-2} \int_{\Re}\int_{\Re} \mathrm{cov}\left(y(s), y(s')\right) ds ds'. \tag{B5}$$

There are also H constraints for the coefficients w_{hi}'s ($h = 1, \ldots, H$ and $i = 1, \ldots, n_h$), as follows:

$$\left.\begin{array}{l} \mu_h \left[\sum_{i=1}^{n_h} w_{hi} - 1\right] = 0 \\ h = 1, \ldots, H \end{array}\right\} \tag{B6}$$

where μ_h's are the Lagrange multipliers.

The optimal weights w_{hi}'s are chosen by minimizing $\sigma_{\Re}^2$ in (B2) subject to the unbiasedness constraints of (B6). In terms of variational analysis [34], this situation is formulated as

$$\left.\begin{array}{l} \frac{\partial}{\partial w_{hi}} \left(\sigma_{\Re}^2 + 2\mu_h \left[\sum_{i=1}^{n_h} w_{hi} - 1\right]\right) = 0 \\ \frac{\partial}{\partial \mu_h} \left(\sigma_{\Re}^2 + 2\mu_h \left[\sum_{i=1}^{n_h} w_{hi} - 1\right]\right) = 0 \end{array}\right\} \tag{B7}$$

where $h = 1, \ldots, H$ and $i = 1, \ldots, n_h$. The solution of the system (B7) leads to the system of (13) for surfaces characterized by stratified nonhomogeneity.

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